39. 1. $\triangle ABC \cong \angle CBA$ (Given)
2. $\overline{AB} \cong \overline{CB}$ (CPCTC)
3. $\triangle ABC$ (Def. of Isosc. ρ)

40. Two sides of a $\triangle$ are $\cong$ if and only if the $\angle$s opp. those sides are $\cong$.

41.

Statements	Reasons
1. $\triangle ABC$ and $\triangle DEF$	1. Given
2. Draw $\overrightarrow{EF}$ so that $FG = CB$.	2. Through any 2 pts. there is exactly 1 line.
3. $\overline{FG} \cong \overline{CB}$	3. Def. of $\cong$ segs.
4. $\overline{AC} \cong \overline{DF}$	4. Given
5. $\angle C$, $\angle F$ are rt. $\angle$s.	5. Given
6. $\overline{DF} \perp \overline{EG}$	6. Def. of $\perp$ lines
7. $\angle DFG$ is rt. $\angle$	7. Def. of rt. $\angle$
8. $\angle DFG \cong \angle C$	8. Rt. $\angle \cong$ Thm.
9. $\triangle ABC \cong \triangle DGF$	9. SAS Steps 3, 8, 4
10. $\overline{DG} \cong \overline{AB}$	10. CPCTC
11. $\overline{AB} \cong \overline{DE}$	11. Given
12. $\overline{DG} \cong \overline{DE}$	12. Trans. Prop. of $\cong$
13. $\angle G \cong \angle E$	13. Isosc. $\triangle$ Thm.
14. $\angle DFG \cong \angle DFE$	14. Rt. $\angle \cong$ Thm.
15. $\triangle DGF \cong \triangle DEF$	15. AAS Steps 13, 14, 12
16. $\triangle ABC \cong \triangle DEF$	16. Trans. Prop. of $\cong$

42. A

$m\angle VUT = m\angle VTU$
$2m\angle VUT + m\angle VTU + m\angle TUV = 180$
$2m\angle VUT + 20 = 180$
$m\angle VUT = 80°$
$m\angle VUR + m\angle VUT = 90$
$m\angle VUR + 80 = 90$
$m\angle VUR = 10°$

43. H

$y + 10 = 3y - 5$
$15 = 2y$
$y = 7\frac{1}{2}$

44. 13.5

$6t - 9 + 4t + 4t = 180$
$14t = 189$
$t = 13.5$

CHALLENGE AND EXTEND

45. It is given that $\overline{JK} \cong \overline{JL}$, $\overline{KM} \cong \overline{KL}$, and $m\angle J = x°$. By the $\triangle$ Sum Thm., $m\angle JKL + m\angle JLK + x° = 180°$. By the Isosc. $\triangle$ Thm., $m\angle JKL = m\angle JLK$. So $2(m\angle JLK) + x° = 180°$. or $m\angle JLK = \left(\frac{180 - x}{2}\right)°$. Since $m\angle KML = m\angle JLK$, $m\angle KML = \left(\frac{180 - x}{2}\right)°$ by the Isosc. $\triangle$ Thm. By the $\triangle$ Sum Thm., $m\angle MKL + m\angle JLK + m\angle KML = 180°$ or $m\angle MKL = 180° - \left(\frac{180 - x}{2}\right)° - \left(\frac{180 - x}{2}\right)°$. Simplifying gives $m\angle MKL = x°$.

46. Let $A = (x, y)$.

$4a^2 = AB^2$
$= x^2 + y^2$
$= AC^2$
$= (x - 2a)^2 + y^2$
$= x^2 - 4ax + 4a^2 + y^2$
$= 4a^2 - 4ax + 4a^2$
$4ax = 4a^2$
$x = a$
$y = \pm\sqrt{4a^2 - x^2}$
$= \pm a\sqrt{3}$
$(x, y) = \left(a, a\sqrt{3}\right)$

47. $(2a, 0)$, $(0, 2b)$, or any pt. on the $\perp$ bisector of $\overline{AB}$.

SPIRAL REVIEW

48. $x^2 + 5x + 4 = 0$
$(x + 4)(x + 1) = 0$
$x = -4$ or -1

49. $x^2 - 4x + 3 = 0$
$(x - 3)(x - 1) = 0$
$x = 3$ or 1

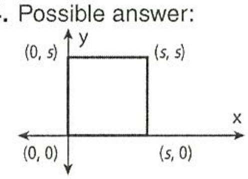
50. $x^2 - 2x + 1 = 0$
$(x - 1)(x - 1) = 0$
$x = 1$

51. $m = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{5 - (-1)}{0 - 2}$
$= \frac{6}{-2} = -3$

52. $m = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{-10 - (-10)}{20 - (-5)} = 0$

53. $m = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{11 - 7}{10 - 4}$
$= \frac{4}{6} = \frac{2}{3}$

54. Possible answer:



#1, 3, 4, 8-12

READY TO GO ON? PAGE 281

1. It is given that $\overline{AC} \cong \overline{BC}$, and $\overline{DC} \cong \overline{DC}$ by Reflex. Prop. of $\cong$. By the Rt. $\angle \cong$ Thm., $\angle ACD \cong \angle BCD$. Therefore, $\triangle ACD \cong \triangle BCD$ by SAS.

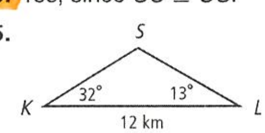
2.

Statements	Reasons
1. $\overline{JK}$ bisects $\angle MJN$.	1. Given
2. $\angle MJK \cong \angle NJK$	2. Def. of $\angle$ bisector
3. $\overline{MJ} \cong \overline{NJ}$	3. Given
4. $\overline{JK} \cong \overline{JK}$	4. Reflex. Prop of $\cong$
5. $\triangle MJK \cong \triangle NJK$	5. SAS Steps 3, 2, 4

3. Yes, since $\overline{SU} \cong \overline{US}$.

4. No; need $\overline{AC} \cong \overline{DB}$.

5.



6. Yes; the $\triangle$ is uniquely determined by ASA.

7.

Statements	Reasons
1. $\overline{CD} \parallel \overline{BE}$ and $\overline{DE} \parallel \overline{CB}$	1. Given
2. $\angle DEC \cong \angle BCE$ and $\angle DCE \cong \angle BEC$	2. Alt. Int. $\angle$s Thm.
3. $\overline{CE} \cong \overline{EC}$	3. Reflex. Prop of $\cong$
4. $\triangle DEC \cong \triangle BCE$	4. ASA Steps 2, 3
5. $\angle D \cong \angle B$	5. CPCTC

8. Check students' drawings; possible answer: vertices at (0, 0), (9, 0), (9, 9), and (0, 9).

9. It is given that $ABCD$ is a rect. M is the mdpt. of $\overline{AB}$, and N is the mdpt. of $\overline{AD}$. Use coords. $A(0, 0)$, $B(2a, 0)$, $C(2a, 2b)$, and $D(0, 2b)$. By Mdpt. Formula, coords. of M are $\left(\frac{0+2a}{2}, \frac{0+0}{2}\right) = (a, 0)$, and coords. of N are $\left(\frac{0+0}{2}, \frac{0+2b}{2}\right) = (0, b)$.
Area of rect. $ABCD = \ell w = (2a)(2b) = 4ab$.
Area of $\triangle AMN = \frac{1}{2}bh = \frac{1}{2}ab$, which is $\frac{1}{8}$ the area of rect. $ABCD$.

10. $m\angle E = m\angle D = 2x°$
$m\angle C + m\angle D + m\angle E = 180$
$5x + 2x + 2x = 180$
$9x = 180$
$x = 20$
$m\angle C = 5x = 100°$

11. By Equiang. $\triangle$ Thm.,
$\overline{RS} \cong \overline{RT} \cong \overline{ST}$
$RS = RT$
$2w + 5 = 8 - 4w$
$6w = 3$
$w = 0.5$
$ST = RS = 2(0.5) + 5 = 6$

12. It is given that isosc. $\triangle JKL$ has coords. $J(0, 0)$, $K(2a, 2b)$, and $L(4a, 0)$. M is mdpt. of $\overline{JK}$, and N is mdpt. of $\overline{KL}$. By Mdpt. Formula, coords. of M are $\left(\frac{0+2a}{2}, \frac{0+2b}{2}\right) = (a, b)$, and coords. of N are $\left(\frac{2a+4a}{2}, \frac{2b+0}{2}\right) = (3a, b)$. By Dist. Formula,
$MK = \sqrt{(2a-a)^2 + (2b-b)^2} = \sqrt{a^2+b^2}$, and
$NK = \sqrt{(2a-3a)^2 + (2b-b)^2} = \sqrt{a^2+b^2}$.
Thus $\overline{MK} \cong \overline{NK}$. So $\triangle KMN$ is isosc. by def. of isosc. $\triangle$.

#1-11, 14, 15, 22-30

STUDY GUIDE: REVIEW, PAGES 284–287

1. isosceles
2. corresponding angles
3. included side

LESSON 4-1

4. equiangular; equilat.
5. obtuse; scalene

LESSON 4-2

6. Think: Use Ext. $\angle$ Thm.
$m\angle N + m\angle P = m(\text{ext. } \angle Q)$
$y + y = 120$
$y = 60$
$m\angle N = y = 60°$

7. Think: Use $\triangle$ $\angle$ Sum Thm.
$m\angle L + m\angle M + m\angle N = 180$
$8x + 2x + 1 + 6x - 1 = 180$
$16x = 180$
$x = 11.25$
$m\angle N = 6x - 1 = 66.5°$

LESSON 4-3

8. $\overline{PR} \cong \overline{XZ}$
9. $\angle Y \cong \angle Q$
10. $m\angle CAD = m\angle ACB$
$2x - 3 = 47$
$2x = 50$
$x = 25$
11. $CD = AB$
$3y + 1 = 15 - 4y$
$7y = 14$
$y = 2$
$CD = 3y + 1 = 7$

LESSON 4-4

12.

Statements	Reasons
1. $\overline{AB} \cong \overline{DE}$, $\overline{DB} \cong \overline{AE}$	1. Given
2. $\overline{DA} \cong \overline{AD}$	2. Reflex. Prop. of $\cong$
3. $\triangle ADB \cong \triangle DAE$	3. SSS Steps 1, 2

13.

Statements	Reasons
1. $\overline{GJ}$ bisects $\overline{FH}$, and $\overline{FH}$ bisects $\overline{GJ}$.	1. Given
2. $\overline{GK} \cong \overline{JK}$, $\overline{FK} \cong \overline{HK}$	2. Def. of seg. bisector
3. $\angle GKF \cong \angle JKH$	3. Vert. $\angle$s Thm.
4. $\triangle FGK \cong \triangle HJK$	4. SAS Steps 2, 3

14. $BC = x^2 + 36 = (-6)^2 + 36 = 72$
$YZ = 2x^2 = 2(-6)^2 = 72 = BC$
$\overline{BC} \cong \overline{YZ}$; $\angle C \cong \angle Z$; $\overline{AC} \cong \overline{XZ}$. So $\triangle ABC \cong \triangle XYZ$ by SAS.

15. $PQ = y - 1 = 25 - 1 = 24$
$QR = y = 25$
$PR = y^2 - (y-1)^2 - 42 = (25)^2 - (24)^2 - 42 = 7$
$\overline{LM} \cong \overline{PQ}$; $\overline{MN} \cong \overline{QR}$; $\overline{LN} \cong \overline{PR}$.
So $\triangle LMN \cong \triangle PQR$ by SSS.

LESSON 4-5

16.

Statements	Reasons
1. C is mdpt. of $\overline{AG}$.	1. Given
2. $\overline{GC} \cong \overline{AC}$	2. Def. of mdpt
3. $\overline{HA} \parallel \overline{GB}$	3. Given
4. $\angle HAC \cong \angle BGC$	4. Alt. Int. $\angle$s Thm.
5. $\angle HCA \cong \angle BCG$	5. Vert. $\angle$s Thm.
6. $\triangle HAC \cong \triangle BGC$	6. ASA Steps 4, 2, 5

17.

Statements	Reasons
1. $\overline{WX} \perp \overline{XZ}$, $\overline{YZ} \perp \overline{ZX}$	1. Given
2. $\angle WXZ$, $\angle YZX$ are rt. $\angle$s.	2. Def. of $\perp$
3. $\triangle WXZ$, $\triangle YZX$ are rt. $\triangle$s.	3. Def. of rt. $\triangle$
4. $\overline{XZ} \cong \overline{ZX}$	4. Reflex. Prop. of $\cong$
5. $\overline{WZ} \cong \overline{YX}$	5. Given
6. $\triangle WZX \cong \triangle YXZ$	6. HL Steps 5, 4

18.

Statements	Reasons
1. $\angle S$, $\angle V$ are rt. $\angle$s.	1. Given
2. $\angle S \cong \angle V$	2. Rt. $\angle \cong$ Thm.
3. $RT = UW$	3. Given
4. $\overline{RT} \cong \overline{UW}$	4. Def. of $\cong$
5. $m\angle T = m\angle W$	5. Given
6. $\angle T \cong \angle W$	6. Def. of $\cong$
7. $\triangle RST \cong \triangle UVW$	7. AAS Steps 2, 6, 4

LESSON 4-6

19.

Statements	Reasons
1. M is mdpt. of $\overline{BD}$.	1. Given
2. $\overline{MB} \cong \overline{DM}$	2. Def. of mdpt.
3. $\overline{BC} \cong \overline{DC}$	3. Given
4. $\overline{CM} \cong \overline{CM}$	4. Reflex. Prop. of $\cong$
5. $\triangle CBM \cong \triangle CDM$	5. SSS Steps 2, 3, 4
6. $\angle 1 \cong \angle 2$	6. CPCTC

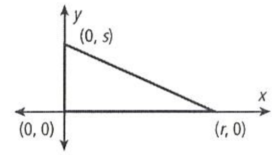
20.

Statements	Reasons
1. $\overline{PQ} \cong \overline{RQ}$	1. Given
2. $\overline{PS} \cong \overline{RS}$	2. Given
3. $\overline{QS} \cong \overline{QS}$	3. Reflex. Prop. of $\cong$
4. $\triangle PQS \cong \triangle RQS$	4. SSS Steps 1, 2, 3
5. $\angle PQS \cong \angle RQS$	5. CPCTC
6. $\overline{QS}$ bisects $\angle PQR$.	6. Def. of $\angle$ bisector

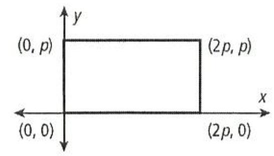
21.

Statements	Reasons
1. H is mdpt. of $\overline{GJ}$, L is mdpt. of $\overline{MK}$.	1. Given
2. $GH = JH$, $ML = KL$	2. Def. of mdpt.
3. $\overline{GH} \cong \overline{JH}$, $\overline{ML} \cong \overline{KL}$	3. Def. of $\cong$
4. $\overline{GJ} \cong \overline{KM}$	4. Given
5. $\overline{GH} \cong \overline{KL}$	5. Div. Prop. of $\cong$
6. $\overline{GM} \cong \overline{KJ}$, $\angle G \cong \angle K$	6. Given
7. $\triangle GMH \cong \triangle KJL$	7. ASA Steps 5, 6
8. $\angle GMH \cong \angle KJL$	8. CPCTC

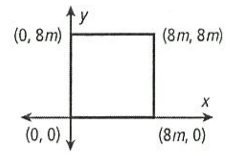
22. Check students' drawings; e.g., (0, 0), (r, 0), (0, s)



23. Check students' drawings; e.g., (0, 0), (2p, 0), (2p, p), (0, p)



24. Check students' drawings; e.g., (0, 0), (8m, 0), (8m, 8m), (0, 8m)



LESSON 4-7

25. Use coords. $A(0, 0)$, $B(2a, 0)$, $C(2a, 2b)$, and $D(0, 2b)$. Then by Mdpt. Formula, the mdpt. coords are $E(a, 0)$, $F(2a, b)$, $G(a, 2b)$, and $H(0, b)$. By Dist. Formula, $EF = \sqrt{(2a - a)^2 + (b - 0)^2} = \sqrt{a^2 + b^2}$, and $GH = \sqrt{(0 - a)^2 + (b - 2b)^2} = \sqrt{a^2 + b^2}$. So $\overline{EF} \cong \overline{GH}$ by def. of $\cong$.

26. Use coords. $P(0, 2b)$, $Q(0, 0)$, and $R(2a, 0)$. By Mdpt. Formula, mdpt. coords are $M(a, b)$. By Dist. Formula, $QM = \sqrt{(a - 0)^2 + (b - 0)^2} = \sqrt{a^2 + b^2}$, $PM = \sqrt{(a - 0)^2 + (b - 2b)^2} = \sqrt{a^2 + b^2}$, and $RM = \sqrt{(2a - a)^2 + (0 - b)^2} = \sqrt{a^2 + b^2}$. So $QM = PM = RM$. By def., M is equidistant from vertices of $\triangle PQR$.

27. In a rt. $\triangle$, $a^2 + b^2 = c^2$.
$\sqrt{(3 - 3)^2 + (5 - 2)^2} = 3$,
$\sqrt{(3 - 2)^2 + (2 - 5)^2} = \sqrt{10}$,
$\sqrt{(2 - 3)^2 + (5 - 5)^2} = 1$, and $3^2 + 1^2 = \left(\sqrt{10}\right)^2$.
Since $9 + 1 = 10$, it is a rt. $\triangle$.

LESSON 4-8

28. Think: Use Equilat. $\triangle$ Thm. and $\triangle$ $\angle$ Sum Thm.
$m\angle K = m\angle L = m\angle M$
$m\angle K + m\angle L + m\angle M = 180$
$3m\angle M = 180$
$3(45 - 3x) = 180$
$-45 = 9x$
$x = -5$

29. Think: Use Conv. of Isosc. $\triangle$ Thm.
$\overline{RS} \cong \overline{RT}$
$RS = RT$
$1.5y = 2y - 4.5$
$4.5 = 0.5y$
$y = 9$
$RS = 1.5y = 13.5$

30. $\overline{AB} \cong \overline{BC}$
$AB = BC$
$x + 5 = 2x - 3$
$8 = x$
Perimeter $= AC + CD + AD$
$= 2AB + CD + CD$
$= 2(x + 5) + 2(2x + 6)$
$= 6x + 22$
$= 6(8) + 22 = 70$ units

CHAPTER TEST, PAGE 288 #1-9, 14-18

1. Rt. △

2. scalene △ ($AC = 4$ by Pythag. Thm)

3. isosc. △ ($AC = BC = 4$)

4. scalene △ ($BD = 4 + 3 = 7$)

5. $m\angle RTP = 2m\angle RTS$
$m\angle RTP + m\angle RTS = 180$
$3m\angle RTS = 180$
$m\angle RTS = 60°$
$m\angle RTS + m\angle R + m\angle S = 180$
$60 + m\angle R + 43 = 180$
$m\angle R = 77°$

6. $\overline{JL} \cong \overline{XZ}$ **7.** $\angle Y \cong \angle K$

8. $\angle L \cong \angle Z$ **9.** $\overline{YZ} \cong \overline{KL}$

10.

Statements	Reasons
1. T is mdpt. of $\overline{PR}$ and $\overline{SQ}$.	1. Given
2. $\overline{PT} \cong \overline{RT}$, $\overline{ST} \cong \overline{QT}$	2. Def. of mdpt.
3. $\angle PTS \cong \angle RTQ$	3. Vert. ∠s Thm.
4. $\triangle PTS \cong \triangle RTQ$	4. SAS Steps 2, 3

11.

Statements	Reasons
1. $\angle H \cong \angle K$	1. Given
2. $\overline{GJ}$ bisects $\angle HGK$.	2. Given
3. $\angle HGJ \cong \angle KGJ$	3. Def. of ∠ bisector
4. $\overline{JG} \cong \overline{JG}$	4. Reflex. Prop. of ≅
5. $\triangle HGJ \cong \triangle KGJ$	5. AAS Steps 1, 3, 4

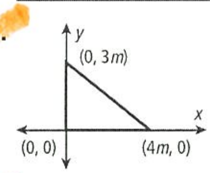
12.

Statements	Reasons
1. $\overline{AB} \perp \overline{AC}$, $\overline{DC} \perp \overline{DB}$	1. Given
2. $\angle BAC$, $\angle CDB$ are rt. ∠s.	2. Def. of ⊥
3. $\triangle ABC$ and $\triangle DCB$ are rt. △s.	3. Def. of rt. △
4. $\overline{AB} \cong \overline{DC}$	4. Given
5. $\overline{BC} \cong \overline{CB}$	5. Reflex. Prop. of ≅
6. $\triangle ABC \cong \triangle DCB$	6. HL Steps 5, 4

13.

Statements	Reasons
1. $\overline{PQ} \parallel \overline{SR}$	1. Given
2. $\angle QPR \cong \angle SRP$	2. Alt. Int. ∠s Thm.
3. $\angle S \cong \angle Q$	3. Given
4. $\overline{PR} \cong \overline{RP}$	4. Reflex. Prop. of ≅
5. $\triangle QPR \cong \triangle SRP$	5. AAS Steps 2, 3, 4
6. $\angle SPR \cong \angle QRP$	6. CPCTC
7. $\overline{PS} \parallel \overline{QR}$	7. Conv. of Alt. Int. ∠s Thm.

14.



15. Use coords. $A(0, 0)$, $B(a, 0)$, $C(a, a)$, and $D(0, a)$. By Dist. Formula,
$AC = \sqrt{(a-0)^2 + (a-0)^2} = a\sqrt{2}$, and
$BD = \sqrt{(0-a)^2 + (a-0)^2} = a\sqrt{2}$. Since
$AC = BD$, $\overline{AC} \cong \overline{BD}$ by def. of ≅.

16. Think: By Equilat. △ Thm., $m\angle F = m\angle G = m\angle H$.
$3m\angle G = 180$
$3(5 - 11y) = 180$
$5 - 11y = 60$
$-11y = 55$
$y = -5$

17. Think: Use △ ∠ Sum and Isosc. △ Thms.
$m\angle P + m\angle Q + m\angle PRQ = 180$
$2(56) + m\angle PRQ = 180$
$m\angle PRQ = 68°$
By Vert. ∠ and Isosc. △ Thms.,
$m\angle T = m\angle SRT = m\angle PRQ = 68°$.
Using △ ∠ Sum and Isosc. Thms.
$m\angle S + m\angle T + m\angle SRT = 180$
$m\angle S + 2(68) = 180$
$m\angle S = 44°$

18. It is given that $\triangle ABC$ is isosc. with coords. $A(2a, 0)$, $B(0, 2b)$, and $C(-2a, 0)$. D is mdpt. of $\overline{AC}$, and E is mdpt. pf $\overline{AB}$. By Mdpt. Formula, coords. of D are $\left(\frac{-2a + 2a}{2}, 0\right) = (0, 0)$, and coords. of E are $\left(\frac{2a + 0}{2}, \frac{0 + 2b}{2}\right) = (a, b)$. By Dist. Formula,
$AE = \sqrt{(a - 2a)^2 + (b - 0)^2} = \sqrt{a^2 + b^2}$, and
$DE = \sqrt{(a - 0)^2 + (b - 0)^2} = \sqrt{a^2 + b^2}$.
Therefore, $\overline{AE} \cong \overline{DE}$ and $\triangle AED$ is isosc.